TDP (General) 1st Semester Exam., 2025
(held in 2026)

MATHEMATICS

(General)

FIRST PAPER

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

1. Answer the following questions : 2×20=40

   (a) State De Moivre's theorem.

   (b) If a, b, c are positive real numbers not all equal, then show that

   $$a^3+b^3+c^3>3abc$$

   (c) If $z_r=\cos\frac{\pi}{2^r}+i\sin\frac{\pi}{2^r}$, then find the value of $z_1 z_2 z_3 \cdots$ to ∞.

   (d) Show that $(n+1)^n>2^n\cdot\lfloor n$, n being a positive integer.

(e) Find the value of the constant d such that the vectors

$$(2\hat{i}-\hat{j}+\hat{k}),\ (\hat{i}+2\hat{j}-3\hat{k}),\ (3\hat{i}+d\hat{j}+5\hat{k})$$

are coplanar.

(f) Find the vector equation of the line passing through the point $(-\hat{i}+2\hat{j}+\hat{k})$ and parallel to the vector $(-3\hat{i}+5\hat{j}+2\hat{k})$.

(g) If $\vec{a}+\vec{b}+\vec{c}=\vec{0}$ and $|\vec{a}|=3$, $|\vec{b}|=5$, $|\vec{c}|=7$, then show that the angle between the vectors $\vec{a}$ and $\vec{b}$ is 60°.

(h) Find a unit vector perpendicular to the plane determined by the vectors $(2\hat{i}-6\hat{j}+3\hat{k})$ and $(4\hat{i}+3\hat{j}+\hat{k})$.

(i) If α and β be two vectors such that $|\vec{\alpha}|=8$, $|\vec{\beta}|=6$ and $\vec{\alpha}\cdot\vec{\beta}=0$, then find the value of $|\vec{\alpha}\times\vec{\beta}|$.

(j) If $\vec{\alpha}=(2,-10,2)$, $\vec{\beta}=(3,1,2)$ and $\vec{\gamma}=(2,1,3)$, then find the vector $\vec{\alpha}\times(\vec{\beta}\times\vec{\gamma})$.

(k) The function $f:A\to R$ be defined by $f(x)=x^2+1$, where $A=\{-2,-1,0,1,2\}$ and R be the set of all real numbers. Find $f(A)$.

(l) Define equivalence relation with an example (justification is not required).

(m) With proper justification, give an example of an integral domain which is not a field.

(n) The set

$$M_2=\left\{\begin{pmatrix} a & b\\ c & d\end{pmatrix} : a, b, c, d \text{ are real numbers}\right\}$$

forms a ring with zero divisors with respect to usual matrix addition and multiplication. Find a zero divisor of this ring.

(o) Write the identity element and inverse element for each element of the set {0, 1, 2} for the operation '∗' from the following composition table :

∗	0	1	2
0	0	1	2
1	1	2	0
2	2	0	1

(p) State Cayley-Hamilton theorem.

(q) Show that if every element of a group $(G, \circ)$ be its own inverse, then it is an Abelian group.

(r) Show by an example that union of two sub-spaces of a vector space may not be a sub-space of that vector space.

(s) Find a basis of $R^3(R)$ containing the vectors (1, 1, 0), (1, 1, 1).

(t) When will a system of non-homogeneous linear equations have (i) a unique solution and (ii) an infinite number of solutions?

GROUP—B

Answer **four** questions, taking **one** from each Unit

UNIT—I

2. (a) If $x^2+y^2=4$, then show that $x^4+y^4+x^{-4}+y^{-4}\geq 8\frac{1}{2}$. 4

(b) Find the general values and the principal value of $(-i)^i$. 3

(c) If $\cos\theta=\frac{1}{2}\left(a+\frac{1}{a}\right)$ and $\cos\phi=\frac{1}{2}\left(b+\frac{1}{b}\right)$, then show that $\cos(\theta+\phi)$ is one of the values of $\frac{1}{2}\left(ab+\frac{1}{ab}\right)$. 3

3. (a) If a, b, c are positive and not all equal, then show that

$$(a+b+c)(ab+bc+ca)>9abc$$ 3

(b) If $\tan\log(x+iy)=a+ib$ and $a^2+b^2\neq 1$, then prove that

$$\tan\log(x^2+y^2)=\frac{2a}{1-a^2-b^2}$$ 4

(c) Using Gregory's series, show that

$$\pi=2\sqrt{3}\left(1-\frac{1}{3\cdot 3}+\frac{1}{5\cdot 3^2}-\frac{1}{7\cdot 3^3}+\cdots\right)$$ 3

UNIT—II

4. (a) Prove that every superset of a linearly dependent set of vectors is linearly dependent. 3

(b) If D is the midpoint of the side BC of a triangle ABC, then show that

$$AB^2+AC^2=2(AD^2+BD^2)$$ 4

(c) Find the torque about the point $(3\hat{i}-\hat{j}+3\hat{k})$ of a force $(4\hat{i}+2\hat{j}+\hat{k})$ passing through the point $(5\hat{i}+2\hat{j}+4\hat{k})$. 3

5. (a) Prove that

$$[\vec{a}+\vec{b},\ \vec{b}+\vec{c},\ \vec{c}+\vec{a}]=2[\vec{a}\ \vec{b}\ \vec{c}]$$ 3

(b) Find the set of vectors reciprocal to the vectors $\hat{i}$, $\hat{i}+\hat{j}$, $\hat{i}+\hat{j}+\hat{k}$. 4

(c) Prove by vector method that the median to the base of an isosceles triangle is perpendicular to the base. 3

UNIT—III

6. (a) Show that the set $S=\{1, 2, 3, 4\}$ forms an Abelian group for the operation 'multiplication modulo 5'. 3

(b) Check whether the relation '*a* divides *b*' in the set of all positive integers, is an equivalence relation or not. 3

(c) Verify that the set of all even integers forms a ring under usual addition and multiplication. 4

7. (a) Prove that the mapping $f: I^+ \to I^+$ defined by $f(x)=x^2$, $x \in I^+$ is one-one into, where I^+ is the set of all positive integers. 3

(b) Show that the set Z of all integers forms a group under the binary operation '$*$' defined by $a*b=a+b+1$; $a, b \in Z$. 4

(c) Prove that every field is an integral domain. 3

UNIT—IV

8. (a) Solve the following system of equations by matrix method : 3

$$x+y+z=8,\ x-y+2z=6,\ 3x+5y-7z=14$$

(b) Show that

$$S=\{(x, y, z) \in R^3 : 3x-4y+z=0\}$$

is a subspace in $R^3(R)$. 3

(c) If $T: R^2 \to R^2$ be a linear transformation whose representation matrix is given by $[T]=\begin{pmatrix} 1/2 & 1 \\ 2/3 & 4 \end{pmatrix}$ associated with the basis $\{(1, 0), (1, 1)\}$, then find $T(x, y)$. 4

9. (a) Find the eigenvalues and eigenvectors of the matrix $A=\begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$. 4

(b) Verify whether $T: R^3 \to R^3$ defined by $T(x, y, z)=(x+y, y+z, z+x)$ is a linear transformation or not. 3

(c) Find the range and kernel of the linear transformation $T: R^2 \to R^3$ given by $T(x, y)=(x, x+y, y)$. 3

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7. (a) Solve, by the method of variation of parameters, the following differential equation : 5

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}=e^x\sin x$$

(b) Solve : 5

$$(x^2y^3+xy)dy=dx$$

UNIT—IV

8. (a) Find the orthogonal trajectories of family of parabolas $y^2=4a(x+a)$, where a is parameter. 5

(b) Solve : 5

$$\frac{dy}{dx}+\frac{4xy}{x^2+1}=\frac{1}{(x^2+1)^3}$$

9. (a) Solve the following differential equation by the method of undetermined coefficients : 5

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=xe^x$$

(b) Solve : 5

$$\frac{d^2y}{dx^2}+4y=x\cos x$$

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TDP (General) 3rd Semester Exam., 2025 (held in 2026)

MATHEMATICS

(General)

THIRD PAPER

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

GROUP—A

1. Answer the following questions : 2×20=40

(a) Find the value of λ for which the equation $\lambda xy-8x+9y-12=0$ represents a pair of straight lines.

(b) Find the value of k if the equation $x^2+ky^2+4xy=0$ represents two coincident lines.

(c) Find the equation of bisectors of the angle between the lines $2x^2-7xy+6y^2=0$.

(d) If the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-k}{2}$ lies exactly on the plane $2x-4y+z=9$, then find the value of k.

(e) Find the angle between the planes $x-4y+8z=0$ and $2x-3y+6z=2$.

(f) Check whether it is possible for a line to make the angles $45^\circ, 60^\circ, 120^\circ$ with the coordinate axes.

(g) Find the polar equation of the circle which is passing through the pole and has the centre at (a, α).

(h) Find the points on the conic $14/r=3-8\cos\theta$ whose radius vector is 2.

(i) Find the centre and radius of the sphere

$$x^2+y^2+z^2-6x+6y+2z=0$$

(j) Find the distance of the point (3, 2, 1) from the line

$$\frac{x-1}{3}=\frac{y}{4}=\frac{z-1}{1}$$

(k) Determine the order and the degree of the differential equation

$$\sqrt{y+\left(\frac{dy}{dx}\right)^2}=1+x^3$$

(l) Solve $p^2y+2px=y$, where p has its usual meaning.

(m) Solve :

$$x\sqrt{y}dx+(1+y)\sqrt{1+x}dy=0$$

(n) Solve :

$$(y^2e^{xy^2}+4x^3)dx+(2xye^{xy^2}-3y^2)dx=0$$

(o) Solve :

$$x^2\frac{d^2y}{dx^2}-x\frac{dy}{dx}+y=0$$

(p) Find the particular integral of

$$(3D^2+2D-8)y=5\cos x;\ D\equiv\frac{d}{dx}$$

(q) Find the particular integral of $(D^2-1)y=xe^{2x}$.

(r) Define orthogonal trajectories.

(s) Find the equation of the curve for which Cartesian subtangent is constant.

(t) Find the value of

$$\frac{1}{D^2-1}\sin^2 x$$

where

$$D\equiv\frac{d}{dx}$$

GROUP—B

Answer **four** questions, taking **one** from each Unit

UNIT—I

2. (a) Reduce the following equation to its normal form and find out the nature of the conic represented by it : 5

$$x^2+y^2+2xy+2x+2y+1=0$$

(b) Show that the area of the triangle formed by the lines $ax^2+2hxy+by^2$ and $lx+my+n=0$ is

$$\frac{n^2\sqrt{h^2-ab}}{(am^2-2hlm+bl^2)}$$ 5

3. (a) Find the equation of the chord of the conic $l/r=1+e\cos\theta$, joining two points whose vectorial angles are $(\alpha-\beta)$ and $(\alpha+\beta)$. 5

(b) Prove that the straight lines joining the origin to the points of intersection of the straight line $lx+my+n=0$ with the ellipse $x^2/a^2+y^2/b^2=1$ are coincident, if $a^2l^2+b^2m^2=n^2$. 5

(Continued)

UNIT—II

4. (a) Show that the straight lines whose direction cosines are given by the equation $2l+2m-n=0$ and $mn+nl+lm=0$ are at right angles. 5

(b) A variable plane which has constant distance P from the origin meets the axes at A, B, C. Show that the locus of the centroid of the tetrahedron $OABC$ is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{16}{p^2}$. 5

5. (a) Find the shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$. 5

(b) Find the equation of the two tangent planes to the sphere $x^2+y^2+z^2-2y-6z+5=0$ which are parallel to the plane $2x+2y-z=0$. 5

UNIT—III

6. (a) Obtain the complete primitive and singular solution of $y=px+\sqrt{1+p^2}$, where the symbol p has the usual meaning. 5

(b) Solve the following equation : 5

$$\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=x^2e^{3x}$$

(Turn Over)

12. *(a)* Evaluate

$$\iint_R [2a^2 - 2a(x+y) - (x^2 + y^2)]\, dx\, dy$$

where R is the circle

$$x^2 + y^2 + 2a(x+y) = 2a^2$$ 5

(b) State and prove Parseval's identity. 5

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TDP (Honours) 5th Semester Exam., 2025 (held in 2026)

MATHEMATICS

(Honours)

FIFTH PAPER

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions

Answer **eight** questions, taking **two** from each Unit

UNIT—I

1. *(a)* State least upper bound axiom for ℝ. Is the set of all rational numbers enjoy this property? Justify your answer. 1½+3½=5

(b) State and prove Cauchy's first limit theorem. 5

2. *(a)* Prove that every convergent sequence is bounded. Is the converse true? Justify. 3+2=5

(b) (i) Define the upper limit and the lower limit of a sequence using inequalities. 1½+1½=3

(ii) Find the upper limit and the lower limit of the sequence $\{x_n\}$, if they exist, where

$$x_n = \begin{cases} 1 & , \text{ if } n=1 \\ 2 & , \text{ if } n \text{ is even} \\ \text{least prime factor of } n, & \text{if } n \text{ is odd} \end{cases}$$ 2

3. (a) Show that $\mathbb{Z}$ is neither bounded above nor bounded below. 4

(b) Discuss the convergence of the sequence $\{x_n\}$, given by

$$x_n = \frac{1}{1!} - \frac{1}{2!} + \cdots + \frac{(-1)^{n+1}}{\lfloor n}$$ 3

(c) Prove that arbitrary intersection of closed sets is closed. 3

UNIT—II

4. (a) Let $f : [a, b] \to R$ be bounded. Show that f is Riemann integrable if and only if for each $\varepsilon > 0$, $\exists$ a partition P of $[a, b]$ such that $U(P, f) - L(P, f) < t$. 5

(b) Discuss the Riemann integrability of the function $f : [0, 1] \to R$, given by

$$f(x) = \begin{cases} \frac{1}{q}, & \text{when } x = \frac{p}{q},\ p, q \in N,\ (p, q) = 1 \\ 0, & \text{when } x \text{ is irrational or zero} \end{cases}$$ 5

5. (a) If f and g are two functions both bounded and integrable on $[a, b]$, then prove that their product fg is also bounded and integrable on $[a, b]$. 5

(b) Let $f : [0, 1] \to R$ be defined by

$$f(x) = \begin{cases} x, & \text{if } x \in Q \cap [0, 1] \\ x^2, & \text{if } x \in (R - Q) \cap [0, 1] \end{cases}$$

Discuss the Riemann integrability of f on $[0, 1]$. 5

6. (a) Prove that the integral function of an integrable function is continuous. 3

(b) State Weierstrass's form of second mean value theorem of integral calculus. 2

(c) If $f, g : [a, b] \to \mathbb{R}$ are bounded functions and P be any partition of $[a, b]$, then with usual notations prove that

(i) $L(P, f + g) \geq L(P, f) + L(P, g)$

(ii) $U(P, f + g) \leq U(P, f) + U(P, g)$ 5

UNIT—III

7. (a) Define beta function and discuss its convergence. 5

(b) Show that

$$\int_0^\infty \frac{\sin x}{x} dx$$

converges but not absolutely. 5

8. (a) Show that

$$\int_0^a \frac{dx}{(a^n - x^n)^{1/n}} = \frac{\pi}{n\sin(\pi / n)}$$ 5

(b) When n is a positive integer, show that

$$\Gamma\left(n+\frac{1}{2}\right) = \frac{(2n)!}{2^{2n}\, n!}\sqrt{\pi}$$ 5

9. (a) Find the volume of the solid formed by the revolution about x-axis, the figure enclosed by the arcs of the parabolas $y^2 = x$ and $x^2 = y$. 5

(b) Find the volume and surface area of solid generated by revolving about y-axis that part of the astroid

$$x = a\cos^3\theta,\ y = a\sin^3\theta$$

that lies in the first quadrant. 5

UNIT—IV

10. (a) Find the condition so that the series

$$\sum_{n=1}^{\infty} \frac{x}{n^p + x^2 + n^q}$$

converges uniformly for all real x. 5

(b) Find the Fourier series for the function f, where

$$f(x) = \begin{cases} x, & \text{when } -\pi \le x < 0 \\ -x, & \text{when } 0 \le x \le \pi \end{cases}$$ 5

11. (a) Find the Fourier series for the function f, defined by

$$f(x) = \begin{cases} x, & \text{for } 0 \le x < 1 \\ 1-x, & \text{for } 1 \le x \le 2 \end{cases}$$

Hence deduce that

$$1 + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$ 5

(b) By changing the order of integration, prove that

$$\int_0^1 dx \int_0^{\sqrt{1-x^2}} \frac{dy}{(1+e^y)\sqrt{1-x^2-y^2}} = \frac{\pi}{2}\log\left(\frac{2e}{1+e}\right)$$ 5

(b) Find the law of force to the pole when the path is $r = a(1-\cos\theta)$ and prove that if F is the force at the apse? and v the velocity there, then $3v^2 = 4aF$. 5

12. (a) A body projected with an initial velocity u_0 at a height h above the surface of the earth becomes a satellite with circular orbit. Show that $u_0 = a\sqrt{\frac{g_0}{a+h}}$ where g_0 is the acceleration due to gravity on the surface of the earth and a is the radius. 5

(b) If a particle describes the curve $r^n = A\cos(n\theta) + B\sin(n\theta)$, then show that it moves under a central force varying inversely as r^{2n+3}. 5

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TDP (Honours) 5th Semester Exam., 2025 (held in 2026)

MATHEMATICS

(Honours)

SIXTH PAPER

Full Marks : 80

Time : 3 hours

The figures in the margin indicate full marks for the questions.

Answer **eight** questions, taking **two** from each Unit.

UNIT—I

1. (a) For any two sets A and B, show that

$$P(A \cap B) \le P(A) \le P(A \cup B) \le P(A) + P(B)$$ 5

(b) Define Poisson distribution with parameter λ. Find the variance of Poisson distribution. 1+4=5

2. (a) If A and B are two events such that $P(A) = \frac{1}{3}$, $P(B) = \frac{3}{4}$ and $P(A \cup B) = \frac{11}{12}$, then find $P(A \cap B)$, $P(A|B)$ and $P(B|A)$. 1+2+2=5

(b) The random variables X and Y are jointly distributed as follows :

$$f(x, y) = \begin{cases} 6x^2y, & 0 < x < 1, 0 < y < 1 \\ 0, & \text{otherwise} \end{cases}$$

Show that $\int_0^1 \int_0^1 f(x, y)\,dxdy = 1$ 5

3. *(a)* Define moment generating function of a random variable X and find it when X is a normal variate with mean m and standard deviation σ. 5

(b) If X is a continuous random variable with p.d.f.

$$f(x) = ce^{-\frac{x}{\sigma}}, \quad x \geq 0, \ \sigma > 0$$

find its mean and variance. 2+3=5

UNIT—II

4. *(a)* Show that the sample variance

$$S^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i - \overline{X})^2$$

is not an unbiased estimator of population variance σ^2. 5

(b) Draw the histogram and frequency polygon to represent the following distribution : 5

Marks obtained	*No. of students*
25–40	20
40–55	35
55–70	25
70–85	14
85–100	6

Total Students = 100

5. *(a)* Derive the sampling distribution of sample mean $\bar{x}$ and sample variance S^2 of random sample of size n from $N(\mu, \sigma^2)$. 5

(b) If $\bar{x}$ denotes the sample mean for a sample size 15 from a normal $(m, 2)$ population, find the probability that $3(\bar{x} - m) \geq 4$.

[Take $\frac{1}{\sqrt{2\pi}}\int_0^{2\cdot58} e^{-\frac{x^2}{2}}\,dx = \cdot4951$] 5

6. *(a)* The mean lifetime of a sample of 100 light bulbs produced by a company is computed to be 1870 hours with a standard deviation of 120 hours. If μ is the mean lifetime of all bulbs; test the

hypothesis $\mu = 1600$ hours against the alternative $\mu \neq 1600$ with $\alpha = 0 \cdot 05$ and 0·01.

[Take Z table value at 2·58 at 1% level and 1·96 at 5% level] 5

(b) A random sample of size 10 is drawn from a normal population with known population variance 35·3. If the sample values are 63, 72, 81, 75, 77, 80, 67, 74, 60, 85, obtain a 95% confidence interval of the population mean.

[Given $\frac{1}{\sqrt{2\pi}} \int_{1 \cdot 96}^{\infty} e^{-\frac{x^2}{2}} dx = 0 \cdot 25$] 5

UNIT—III

7. (a) Prove that Kronecker delta δ^i_j is a mixed tensor of type (1, 1). 5

(b) If $A^{ij} (\neq 0)$ are components of a contra-variant tensor of rank 2 such that $bA^{ij} + cA^{ji} = 0$, where b and c are non-zero scalars, show that either $b = c$ and A^{ij} is skew-symmetric or $b = -c$ and A^{ij} is symmetric. 5

8. (a) Show that $\left\{ \begin{matrix} i \\ ij \end{matrix} \right\} = \frac{\partial}{\partial x^j} \log \sqrt{g}$, where $g = |g_{ij}| > 0$. 5

(b) If A_i are components of a covariant vector, then show that

$$\text{curl } A_i = \frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}$$ 5

9. (a) Show that $\frac{\partial g_{ij}}{\partial x^k} - \frac{\partial g_{jk}}{\partial x^i} = [jk, i] - [ij, k]$. 5

(b) If $B^i_{jk} = \delta^i_j \frac{\partial \phi}{\partial x^k} - \delta^i_k \frac{\partial \phi}{\partial x^j}$, where ϕ is invariant, prove that $g^{jk} B^i_{jk} = 0$. 5

UNIT—IV

10. (a) Find the expressions for radial and cross-radial acceleration of a particle moving in a plane curve. 2+3=5

(b) A particle of mass m moves on a straight line under an attraction $mn^2 x$ towards a point O on the line where x is the distance from O. If $x = a$ and $\frac{dx}{dt} = u$ when $t = 0$, find the period of oscillation and the amplitude. 5

11. (a) In a central orbit with usual notations, prove that $v^2 = h^2 \left[u^2 + \left(\frac{du}{d\phi} \right)^2 \right]$. 5